UNIVERSIDADE DE SÃO PAULO

Instituto de Ciências Matemáticas e de Computação

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Decomposition methods for the lot sizing and cutting stock problems in paper industries*

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Abstract

In this paper, we discuss the one-dimensional cutting stock problem coupled with the lot sizing problem in the context of paper industries. The production process in paper mill industries consists in producing raw materials characterised by rolls of paper, and cutting them in smaller rolls according to the demand of customers. Typically, both problems are tackled in sequence, but if the decisions of the cutting patterns and the production of rolls are made together, it can result in better resource management. We investigate Dantzig-Wolfe decompositions and develop column generation techniques to obtain upper and lower bounds for the integrated problem. First, we analyse the classical column generation method for the cutting stock problem built-in the coupled problem. Second, we propose the machine decomposition that is compared with the classical period decomposition for the lot sizing problem. The machine decomposition and the period decomposition provide the same lower bound, that is well-known to be better than the classical lot sizing model. However, the column generation heuristic for the problem without decomposing the lot sizing problem presented better feasible solutions with low computational effort for instances of the literature.

Keywords: Cutting problem; Lot sizing problem; Dantzig-Wolfe decomposition; Column generation method.

1 Introduction

The one-dimensional cutting stock problem (1CSP) and the lot sizing problem (LSP) have been extensively studied in the past few decades. In the 1CSP, we are given a set of objects to be cut into required items, where only one dimension of items and of objects is relevant. On the other hand, the LSP determines how much to produce of a set of items in order to satisfy the required

*Technical Report

demand in each period of a planning horizon without exceeding the capacity constraints. Both problems are encountered in paper, steel, glass and furniture industries. Despite they are related to the same practical applications, there are only few papers in the literature addressing the interaction between these problems.

In this paper, we discuss the 1CSP integrated with the LSP in paper industries (1CSP-LSP). The production process consists in producing raw materials, i.e., rolls of paper, and cutting them into final items in order to meet the demand of customers. Moreover, the required items have different grades and different lengths. The production of raw materials can vary from industry to industry, where the pulp mill can be bought or produced by the industry. The pulp and different types of recycled papers are processed in the paper machine, where the industry can have one or more machines. After the production of the paper rolls of different grades, two types of decisions must be considered: cut the rolls into the demanded items and store items and rolls to the next period in the planning horizon.

The LSP is related to the production of raw materials. It is necessary to determine which grades will be produced in each paper machine with limited capacity during each period of the planning horizon. The grade of the rolls must meet the grade of the demanded items. In addition, there are setup times and setup costs to change the production of rolls with different grades. Then, the LSP involved in the coupled problem tackled in this paper is the capacitated lot sizing problem in parallel machines with setup times and setup costs (CLSPP). In each period of the planning horizon, the 1CSP determines the number of required items to be cut from the rolls of paper. The objective in the CLSPP is to minimise the production costs, the setup costs, and the rolls held at the end of each period. In the 1CSP, the objective is to minimise the waste of paper incurred during the cutting phase and the storage of items. The integration 1CSP-CLSPP occurs by the fact that it is necessary to produce an enough number of rolls to be cut that can attend the demanded items in each period of the planning horizon.

The aim of this paper is to analyse some mixed integer programming models for the 1CSP-CLSPP. The first mathematical model (item-oriented model) integrates the classical CLSPP formulation and the mathematical formulation of Kantorovich (1960) for the 1CSP. In the second model (pattern-oriented model), the 1CPS is modelled according to Gilmore and Gomory (1961), while it keeps the initial CLSPP formulation of the first model. In the third model (machine decomposition model), we consider the 1CSP model written according to Gilmore and Gomory's approach and propose the machine Dantzig-Wolfe decomposition for the CLSPP. The period Dantzig-Wolfe decomposition is also considered, but briefly described.

To solve the proposed models, we implemented column generation methods. First, lower bounds are determined by solving the linear relaxation of the models using the column generation methods. Then, we obtain feasible solutions (upper bounds) by solving the resultant restricted and integer master problem. We evaluate the performance of the methods using instances from the literature and instances based on a real application. Computational results show that solving pattern-oriented model by CPLEX 12.6 provides high-quality solutions in a low computational time.

The main contributions of this paper are: (i) proposals of mathematical formulations for the 1CSP-CLSPP; and (ii) comparison of lower and upper bounds obtained by solving the models. The main reason for the proposal of those decompositions for the 1CSP-CLSPP it is the development of tighter lower bounds for the coupled problem. In a previous paper in the literature, no lower bounds were determined for the problem, where a mathematical formulation and Lagrangean heuristics were developed.

The remainder of this paper is the following. In Section 2, we review some papers in the literature focused on the integration of two or more problems in paper mill manufacturings. In Section 3, we present the mathematical models for the 1CSP-CLSPP. Computational results are reported in Section 4. In Section 5, we draw some conclusions.

2 Literature review

The production process in paper industries is complex and can involve many problems, such as production planning, scheduling problem, transportation problem, one- and two-dimensional cutting stock problems. There are only few papers in the literature that solve the integration of two or more of those problems. The planning and scheduling problems involve decisions about how much paper will be produced in one or more machines during a planning horizon and the sequence of the paper types in each machine. Usually there are setup cost and setup times for changing one type of paper to another one, where a paper type can be characterised by its grade and its colour. After the production of paper rolls, the rolls are cut into smaller ones in the way that the waste of material is minimised, which characterise the one-dimensional cutting stock problem. The smaller rolls can be demanded items or intermediate products. When the demanded items are sheets, a two-dimensional cutting stock problem is defined. Finally, the final items are delivered to customers.

Krichagina et al. (1998) combined the production scheduling and the cutting stock problem in a paper industry that produces sheets of different sizes. The production scheduling problem involves inventory decisions such as the production and the inventory levels of items in order to determine a shutdown planning for the paper machine. The objective is to minimise backorder, holding costs, shutdowns and paper waste. The shutdowns in a paper machine are allowed since the machine capacity is slighty higher than the demand. To solve the problem, they proposed a linear programming model to decide a set of cutting configurations to be used in a Brownian analysis that finds a dynamic scheduling.

Keskinocak et al. (2002) developed a support system for paper mill industries of different locations. Their support system considers the distribution of the demand to paper mills, the scheduling of paper machines in each paper mill, the cutting of rolls into smaller ones (1CSP), and the transportation of the demand to customers. To solve the integration of these problems they use a series of algorithms in an asynchronous team framework.

Menon and Schrage (2002) integrated the 1CSP with the assignment problem of rolls to identical cutting machines. They explored the dual-angular structure of the problem to decompose it and to obtain tight bounds for the subproblems that are used to solve a relaxation of the master problem.

Respicio et al. (2002) also presented a support system for a paper mill company that allows the interaction with the decision makers. They integrated the capacity planning and scheduling of the paper machine with the 1CSP. The problem is decomposed into two subproblems. The first one defines the rolls to be produced over a discrete planning horizon, and the second one defines the machine batching and scheduling. To define the cutting patterns, they used an integer programming model based on Gilmore and Gomory (1961). The linear relaxation of the model is solved by the column generation method and a feasible solution is obtained by rounding the optimal linear relaxation solution. When the paper machine capacity is not completely used, the system compute the forecast demand based on the records over 10 years. In addition, they accept lost of demand if the machine capacity is not enough to produce all requested items.

Correia et al. (2004) integrated the production planning and the cutting stock problem, where they focus mainly on the generation of cutting patterns. The planning production involves the decisions of how much paper must be produced to attend the demand. Due to technical characteristics, the rolls are cut into two stages: the first stage of cuts generates intermediate rolls, and the second one generates the final rolls. As the demanded items can be rolls or sheets, some rolls need to be cut according to a two-dimensional cutting pattern. To solve the problem, some cutting patterns are enumerated and used as columns in a production planning model.

Poltroniere et al. (2008) jointed the 1CSP and the CLSPP to paper industry. The problem consists in determining cutting patterns, deciding their usage during a planning horizon and defining a production scheduling for parallel paper machines. The objective is to minimise holding costs of rolls and final items, production and setup costs, and paper waste occurred during the cutting process. They presented an integer programming formulation and proposed two decomposition heuristics based on Lagrangian relaxation to solve the problem.

Correia et al. (2012) extended the problem described in Correia et al. (2004) by considering multiple machines for the paper production. To solve the problem they used the model proposed in Menon and Schrage (2002) to combine the assignment and cutting problems. The grade sequencing in each paper machine was defined according to the travelling salesman formulation. However, the capacity constraint of the machines was ignored. In an attempt to ensure a feasible solution, they used a re-assignment heuristic.

In Santos and Almada-Lobo (2012), it is proposed a mathematical formulation that integrates the pulp and paper mill production, and the production of electricity using the black liquor obtained during the pulp production. The planning and scheduling problem is modelled as a multi-stage lot sizing and scheduling problem with sequence dependent setup in the paper machine.

Kim et al. (2014) presented a mathematical formulation for one-dimensional two-staged cutting stock problem, where rolls of paper are cut into auxiliary rolls. To determine the auxiliary rolls they define the subpatterns concept, which means that a subpattern is composed by one or more auxiliary rolls of the same order. After the one-dimensional cut, the subpatterns can be cut into sheets. In the mathematical formulation, all subpatterns and patterns are known a priori. In order to solve the problem, they proposed a heuristic to generate the subpatterns and the patterns, and a heuristic to select the patterns.

The integration of cutting stock problem and production planning in other industrial practises has also been studied in the literature such as in furniture (Gramani and França, 2006; Gramani et al., 2009, 2011; Silva et al., 2014), clothing (Farley, 1988), copper (Hendry et al., 1996) and wood processing industries (Reinders, 1992), steel trucks manufacturing (Nonås and Thorstenson, 2000, 2008) and gear belts production (Arbib and Marinelli, 2005).

3 Problem definition and mathematical formulations for the 1CSP-CLSPP

In the 1CSP-CLSPP, we have to attend the demand in a planning horizon with T periods. The items are rolls of papers with different lengths and classified into K different grades. M machines are available to produce bigger rolls of papers (objects) with length given by b_{km}, $k = 1, ..., K$, $m = 1, ..., M$. Hence, the objects must be cut into the demanded items of length given by l_i ($l_i \leq b_{km}$). There is a time consuming to produce the objects and a setup time for changing the production of objects with different grades, where each machine has a capacity time. The problem consists in determining the production planning of the objects without exceeding the machine capacity. In addition, items and objects can be hold from one period to another. The objective is to minimise production and setup costs, inventory costs of items and of objects, and waste cost incurred during the cutting process.

In this section, we present three mathematical formulations for the 1CSP-CLSPP: item-oriented model; pattern-oriented model; and machine decomposition model. For the mathematical formulations, we introduce the following sets, parameters and variables that are common for all models.

Sets:

- $\mathcal{T} = \{1, ..., T\}$: set of periods in the planning horizon;
- $\mathcal{M} = \{1, ..., M\}$: set of machines;
- $\mathcal{K} = \{1, ..., K\}$: set of object grade types;
- $\mathcal{N} = \{n_0 + 1, ...n_1, n_1 + 1, ..., n_2, ..., n_{k-1} + 1, n_K\}$: set of items $n_0 = 0$ and $n_K = n$. The set of items that must be cut from object of type k is given by $\mathcal{N}_k = \{n_{k-1} + 1, ..., n_k\}$, $\forall k \in \mathcal{K}$.
- $\bar{\mathcal{N}}^t_{km} = \{1, ..., N^t_{km}\}$: set of cutting patterns for object of type k produced in machine m in period t, $\forall k \in \mathcal{K}$, $\forall m \in \mathcal{M}$, $\forall t \in \mathcal{T}$;

Parameters:

- c_{km}: production cost of object type k in machine m, $\forall k \in \mathcal{K}$, $\forall m \in \mathcal{M}$;
- $\bar{h}_{km}$: holding cost of object of type k produced in machine m at the end of each period, $\forall k \in \mathcal{K}$, $\forall m \in \mathcal{M}$;
- h_i: holding cost of item of type i at the end of each period, $\forall i \in \mathcal{N}$;
- l_i: length of item of type i, $\forall i \in \mathcal{N}$;
- d_i^t: demand of items of type i in period t, $\forall i \in \mathcal{N}$, $\forall t \in \mathcal{T}$;
- b_{km}: length of object of type k produced in machine m, $\forall k \in \mathcal{K}$, $\forall m \in \mathcal{M}$;
- Cap_m^t: capacity of machine m in period t, $\forall m \in \mathcal{M}$, $\forall t \in \mathcal{T}$;
- st_{km}: setup time of machine m to produce object of type k, $\forall k \in \mathcal{K}$, $\forall m \in \mathcal{M}$;
- stc_{km}: setup cost of machine m to produce object of type k, $\forall k \in \mathcal{K}$, $\forall m \in \mathcal{M}$;
- vt_{km}: production time of object k in machine m, $\forall k \in \mathcal{K}$, $\forall m \in \mathcal{M}$;
- cl_{km}: paper waste cost of object type k produced in machine m, $\forall k \in \mathcal{K}$, $\forall m \in \mathcal{M}$;
- M_{km}^t: a big number defined as $M_{km}^t = \frac{C_{mt}}{vt_{km}}$, $\forall k \in \mathcal{K}$, $\forall m \in \mathcal{M}$, $\forall t \in \mathcal{T}$.

Decision variables:

- e_{km}^t: number of objects of type k produced in machine m hold at the end of period t, $\forall k \in \mathcal{K}$, $\forall m \in \mathcal{M}$, $\forall t \in \mathcal{T}$;
- s_i^t: holding quantity of item of type i at the end of period t, $\forall i \in \mathcal{N}$, $\forall t \in \mathcal{T}$;
- r_{km}^t: number of objects of type k produced in machine m in period t, $k \in \mathcal{K}$, $m \in \mathcal{M}$, $t \in \mathcal{T}$;
- z_{km}^t: 1 if object of type k is produced in machine m in period t, 0 otherwise, $k \in \mathcal{K}$, $m \in \mathcal{M}$, $t \in \mathcal{T}$.

3.1 Item-oriented model

In the item-oriented model (IOM), the 1CSP is modelled according to Kantorovich (1960) that is joined with the classical CLSPP formulation. It is well-known that the use of a commercial solvers to solve Kantorovich's model is impractical for large problems and it can be even impossible to store the mathematical model on a computer. Then, the IOM is described just to illustrate a sequence of Dantzig-Wolfe decompositions that can be applied for the 1CLSP-CLPP. The Dantizg-Wolfe decomposition of the Kantorovich's model is equivalent to the model proposed by Gilmore and Gomory (1961), that is introduced in the next section.

For the 1CSP it is necessary to define an upper bound on the number of cutting patterns used to attend the demand represented by N_{km}^t for each k, m, t. Each cutting pattern j ($j = 1, ..., N_{km}^t$) is associated with an integer variable and a knapsack constraint to ensure that if a pattern is used, then it is feasible. Thus, a good estimation of N_{km}^t reduces the computational effort to solve and to store the mathematical model.

Let us consider the following additional variables.

- u^t_{jkm}: 1 if object j of type k is produced in machine m and is cut in period t, and 0 otherwise, $j \in \bar{\mathcal{N}}^t_{km}$, $k \in \mathcal{K}$, $m \in \mathcal{M}$, $t \in \mathcal{T}$;
- x^t_{ijm}: number of items of type i cut from object j of type k produced in machine m in period t, $i \in \mathcal{N}_k$, $k \in \mathcal{K}$, $m \in \mathcal{M}$, $t \in \mathcal{T}$.

Then, the item-oriented model is given as follows.

$$z^{IOM} = \text{minimise} \quad \sum_{t\in\mathcal{T}}\sum_{i\in\mathcal{N}} h_i s^t_i + \sum_{t\in\mathcal{T}}\sum_{m\in\mathcal{M}}\sum_{k\in\mathcal{K}} \bar{h}_{km} e^t_{km} + \sum_{t\in\mathcal{T}}\sum_{m\in\mathcal{M}}\sum_{k\in\mathcal{K}} stc_{km} z^t_{km} + \sum_{t\in\mathcal{T}}\sum_{m\in\mathcal{M}}\sum_{k\in\mathcal{K}} c_{km} r^t_{km} + \sum_{t\in\mathcal{T}}\sum_{m\in\mathcal{M}}\sum_{k\in\mathcal{K}}\sum_{j\in\bar{\mathcal{N}}^t_{km}} cl_{km}(b_{km}u^t_{jkm} - \sum_{i\in\mathcal{N}_k} l_i x^t_{ijm}) \quad (1)$$

$$\text{s. t.} \sum_{m\in\mathcal{M}}\sum_{j\in\bar{\mathcal{N}}^t_{km}} x^t_{ijm} + s^{t-1}_i = d^t_i + s^t_i \qquad \forall i \in \mathcal{N}, \forall k \in \mathcal{K}, \forall t \in \mathcal{T} \quad (2)$$

$$\sum_{i\in\mathcal{N}_k} l_i x^t_{ijm} \leq b_{km} u^t_{jkm}, \qquad j \in \bar{\mathcal{N}}^t_{km} \forall k \in \mathcal{K}, \forall m \in \mathcal{M}, \forall t \in \mathcal{T} \quad (3)$$

$$r^t_{km} + e^{t-1}_{km} - e^t_{km} = \sum_{j\in\bar{\mathcal{N}}^t_{km}} u^t_{jkm} \qquad \forall k \in \mathcal{K}, \forall m \in \mathcal{M}, \forall t \in \mathcal{T} \quad (4)$$

$$\sum_{k\in\mathcal{K}} (st_{km} z^t_{km} + vt_{km} r^t_{km}) \leq Cap^t_m \qquad \forall m \in \mathcal{M}, \forall t \in \mathcal{T} \quad (5)$$

$$r^t_{km} \leq M^t_{km} z^t_{km} \qquad \forall k \in \mathcal{K}, \forall m \in \mathcal{M}, t \in \mathcal{T} \quad (6)$$

$$s^0_i = 0, e^0_{km} = 0, \qquad \forall i \in \mathcal{N}, \forall k \in \mathcal{K}, \forall m \in \mathcal{M} \quad (7)$$

$$x^t_{ijm}, r^t_{km}, e^t_{km}, s^t_i \in \mathbb{Z}_+, u^t_{jkm} \in \{0,1\}, \quad \forall i \in \mathcal{N}, j \in \bar{\mathcal{N}}^t_{km}, \forall k \in \mathcal{K}, \forall m \in \mathcal{M}, \forall t \in \mathcal{T}. \quad (8)$$

The objective function (1) minimises the sum of holding cost of items (h_i), the holding cost of the objects ($\bar{h}_{km}$), the setup costs (stc_{km}), the production cost of the objects (c_{km}), and the waste cost occurred in the cutting phase (cl_{km}). Constraints (2) are the operational demand constraints, where the inventory demand of items is considered. Constraints (3) determine feasible cutting patterns for the objects. The tactical demand constraints are represented in (4), they couple the cutting stock problem with the lot sizing problem by taking into account the total demand of objects for the cutting phase and their production and holding stock. The production capacity constraint of the machines is given in (5). If an object is produced in period t in machine m, constraints (6) force its respective setup variable (z^t_{km}) to one. Without loss of generality, we assume in constraints (7) that there is no initial inventories. Constraints (8) define the domain of the variables.

The model of Kantorovich (1960) is well known by its weak linear relaxation, which is not different for the item-oriented model. Stronger relaxations can be obtained by the relaxation of the models described in the following sections.

3.2 Pattern-oriented model

The pattern-oriented model (POM) considers the 1CSP model proposed by Gilmore and Gomory (1961). This model can be seen as a Dantzig-Wolfe decomposition of the model of Kantorovich (1960). For further details, see Vance (1998) and Ben Amor and Valério de Carvalho (2005). In this model, the cutting patterns are considered known, i.e., all the possible solutions of constraints (9) - (10) are known:

$$\textstyle\sum_{i=n_{k-1}+1}^{n_k} l_i a_i \leq b_{km}, \tag{9}$$

$$a_i \in \mathbb{Z}_+,\ i = n_{k-1}+1, ..., n_k. \tag{10}$$

Let a_{ijm} be the number of items of type i cut from pattern j, $i \in \mathcal{N}_k$, $j \in \bar{\mathcal{N}}^t_{km}$, $m \in \mathcal{M}$, $k \in \mathcal{K}$, $t \in \mathcal{T}$. The variable y^t_{jkm} is the number objects of type k produced in machine m, in period t, and cut according to pattern j, $j \in \bar{\mathcal{N}}^t_{km}$, $k \in \mathcal{K}$, $m \in \mathcal{M}$, $t \in \mathcal{T}$.

Therefore, the master problem can be rewritten as follows.

$$z^{POM} = \text{minimise} \quad \sum_{t\in\mathcal{T}}\sum_{i\in\mathcal{N}} h_i s^t_i + \sum_{t\in\mathcal{T}}\sum_{m\in\mathcal{M}}\sum_{k\in\mathcal{K}} \bar{h}_{km} e^t_{km} + \sum_{t\in\mathcal{T}}\sum_{m\in\mathcal{M}}\sum_{k\in\mathcal{K}} stc_{km} z^t_{km} + \sum_{t\in\mathcal{T}}\sum_{m\in\mathcal{M}}\sum_{k\in\mathcal{K}} c^t_{km} r^t_{km} + \sum_{t\in\mathcal{T}}\sum_{m\in\mathcal{M}}\sum_{k\in\mathcal{K}}\sum_{j\in\bar{\mathcal{N}}^t_{km}} cl_{km}(b_{km} - \sum_{i\in\mathcal{N}_k} l_i a_{ij}) y^t_{jkm} \tag{11}$$

$$\text{s. t.} \sum_{m\in\mathcal{M}}\sum_{j\in\bar{\mathcal{N}}^t_{km}} a_{ijm} y^t_{jkm} + s^{t-1}_i = d^t_i + s^t_i \qquad \forall i \in \mathcal{N}_k, \forall k \in \mathcal{K}, \forall t \in \mathcal{T} \tag{12}$$

$$r^t_{km} + e^{t-1}_{km} - e^t_{km} = \sum_{j\in\bar{\mathcal{N}}^t_{km}} y^t_{jkm} \qquad \forall k \in \mathcal{K}, \forall m \in \mathcal{M}, \forall t \in \mathcal{T} \tag{13}$$

$$\sum_{k\in\mathcal{K}} (st_{km} z^t_{km} + vt_{km} r^t_{km}) \leq Cap^t_m \qquad \forall m \in \mathcal{M}, \forall t \in \mathcal{T} \tag{14}$$

$$r^t_{km} \leq M^t_{km} z^t_{km} \qquad \forall k \in \mathcal{K}, \forall m \in \mathcal{M}, \forall t \in \mathcal{T} \tag{15}$$

$$s^0_i = 0, e^0_{km} = 0 \qquad \forall i \in \mathcal{N}, \forall k \in \mathcal{K}, \forall m \in \mathcal{M} \tag{16}$$

$$y^t_{jkm}, r^t_{km}, e^t_k, s^t_i \in \mathbb{Z}_+, z^t_{km} \in \{0,1\} \qquad \forall i \in \mathcal{N}, j \in \bar{\mathcal{N}}^t_{km}, \forall k \in \mathcal{K}, \forall m \in \mathcal{M}, \forall t \in \mathcal{T}. \tag{17}$$

Constraints (12), (13), (14), (15) are the operational demand, the tactical demand, the capacity, and the setup constraints, respectively. These constraints are equivalent to constraints (2), (4), (5), and (6), respectively. The objective function (11) minimises the inventory cost for the objects, the cost for holding final items, the setup cost, the production cost and the waste cost. Constraints (16) set the initial inventories to zero. Constraints (17) define the domain of the variables.

This model is a variation of that one proposed by Poltroniere et al. (2008). The authors considered additional inventory balancing constraints for the objects, where the weight of the objects must meet the weight of the demanded items. In addition, the demand in these constraints is increased by a waste estimative. It is important to highlight that this estimative is necessary in their model since the problem was solved by heuristics that alternate between the solutions of the 1CSP and the CLSPP.

As it is impossible to generate all cutting patterns, they are considered implicitly. To generate cutting patterns for the master problem, the linear relaxation of the master problem with a subset of columns is solved by a column generation method. In order to determine the pricing problem, let us define π_i^t the dual variables associated with constraints (12), $\forall i \in \mathcal{N}$, and $\forall t \in \mathcal{T}$, and γ_{km}^t the dual variables associated with constraints (13), $\forall k \in \mathcal{K}$, $\forall m \in \mathcal{M}$, $\forall t \in \mathcal{T}$The reduced cost is given by:

$$rc1_{km}^t = cl_{km}b_{km} - \sum_{i \in \mathcal{N}_k} (\pi_i^t + cl_{km}l_i)a_i + \sigma_{km}^t, \tag{18}$$

that must be minimised. Then, the subproblem to be solved for the object of type k produced in machine m and in period t is formulated as follows.

$$\text{maximise} \quad \sum_{i \in \mathcal{N}_k} (\pi_i^t + cl_{km}l_i)a_i \tag{19}$$

$$\text{s. t.} \quad \sum_{i=n_{k-1}+1}^{n_k} l_i a_i \leq b_{km} \tag{20}$$

$$a_i \in \mathbb{Z}_+, i \in \mathcal{N}_k. \tag{21}$$

In the column generation method, we have $K \times M \times T$ subproblems. If $rc1_{km}^t \geq 0$, for all k, m and t, then an optimal solution of the linear relaxation of the master problem (11) - (17) is found.

3.3 Machine decomposition model

In this section, we propose the machine Dantzig-Wolfe decomposition for the lot sizing constraints. The development of period decomposition is very similar to the machine decomposition, then it is omitted here for brevity. Model (11) - (17) is decomposed in M subproblems and the master problem is rewritten, where each subproblem determines the production plan for one machine. For this, let us represent

$$CM_m = conv\{\mathtt{z}_m \in \{0,1\}^{\mathcal{K}\times\mathcal{T}}, \mathtt{r}_m \in \mathbb{Z}_+^{\mathcal{K}\times\mathcal{T}} : \sum_{k \in \mathcal{K}} (st_{km}z_{km}^t + vt_{km}r_{km}^t) \leq Cap_m^t, t \in \mathcal{T}; r_{km}^t \leq M_{km}^t z_{km}^t,\ k \in \mathcal{K}, t \in \mathcal{T}\}$$

as the convex hull of the feasible solutions of constraints (14) and (15) for machine m. In this case, the master problem chooses the production plan for each machine and the cutting patterns according to the pattern-oriented model. Then, we have $K \times M \times T$ subproblems related to the pattern generation and M subproblems related to the production plan. For the machine decomposition model (MDM), we define the following parameters and variables.
Parameters:

- $\mathcal{P}_m$: set of convex hull's vertices, i.e., a set of production plans related to machine m, $m \in \mathcal{M}$;
- $r_{km}^{tp_m}$: number of objects of type k produced in machine m in period t according to production plan p_m, $k \in \mathcal{K}$, $t \in \mathcal{T}$, $p_m \in \mathcal{P}_m$, $m \in \mathcal{M}$;
- $z_{km}^{tp_m}$: 1 if object of type k is produced in machine m, in period t according to production plan p_m, 0 otherwise, $k \in \mathcal{K}$, $t \in \mathcal{T}$, $p_m \in \mathcal{P}_m$, $m \in \mathcal{M}$.

Variables:

- $\bar{r}_m^{p_m}$: 1 if the plan p_m of machine m is used, and 0 otherwise, $p_m \in \mathcal{P}_m$, $m \in \mathcal{M}$.

Then, the variables r_{km}^t e z_{km}^t can be rewritten as the convex combination of the vertices of CM_m, $\forall m \in \mathcal{M}$:

$$r_{km}^t = \sum_{p_m \in \mathcal{P}_m} r_{km}^{tp_m} \bar{r}_m^{p_m} \quad \forall k \in \mathcal{K}, \forall t \in \mathcal{T}, \tag{22}$$

$$z_{km}^t = \sum_{p_m \in \mathcal{P}_m} z_{km}^{tp_m} \bar{r}_m^{p_m} \quad \forall k \in \mathcal{K}, \forall t \in \mathcal{T} \tag{23}$$

$$\sum_{p_m \in \mathcal{P}_m} \bar{r}_m^{p_m} \leq 1 \tag{24}$$

$$\bar{r}_m^{p_m} \in \mathbb{Z}_+ \qquad \forall p_m \in \mathcal{P}_m. \tag{25}$$

We denote the cost of a extreme point p_m by:

$$c_m^{p_m} = \sum_{t \in \mathcal{T}} \sum_{k \in \mathcal{K}} stc_{km} z_{km}^t + \sum_{t \in \mathcal{T}} \sum_{k \in \mathcal{K}} c_{km}^t r_{km}^t. \tag{26}$$

Therefore, the master problem can be formulated as follows.

$$z^{MDM} = \text{minimise} \quad \sum_{t \in \mathcal{T}} \sum_{i \in \mathcal{N}} h_i s_i^t + \sum_{t \in \mathcal{T}} \sum_{m \in \mathcal{M}} \sum_{k \in \mathcal{K}} \bar{h}_{km} e_{km}^t + \sum_{m \in \mathcal{M}} \sum_{p_m \in \mathcal{P}_m} c_m^{p_m} \bar{r}_m^{p_m} + \sum_{t \in \mathcal{T}} \sum_{m \in \mathcal{M}} \sum_{k \in \mathcal{K}} \sum_{j \in \bar{\mathcal{N}}_{km}^t} cl_{km}(b_{km} - \sum_{i \in \mathcal{N}_k} l_i a_{ij}) y_{jkm}^t \tag{27}$$

$$
\begin{aligned}
\text{s. t.} \sum_{m\in\mathcal{M}} \sum_{j\in\bar{\mathcal{N}}_{km}^t} a_{ijm} y_{jkm}^t + s_i^{t-1} - s_i^t = d_i^t \qquad & \forall i \in \mathcal{N}_k, \forall k \in \mathcal{K}, \forall t \in \mathcal{T} \qquad (28)\\
\sum_{p_m\in\mathcal{P}_m} r_{km}^{tp_m} \bar{r}_m^{p_m} + e_{km}^{t-1} - e_{km}^t = \sum_{j\in\bar{\mathcal{N}}_{km}^t} y_{jkm}^t \qquad & \forall k \in \mathcal{K}, \forall m \in \mathcal{M}, \forall t \in \mathcal{T} \qquad (29)\\
\sum_{p_m\in\mathcal{P}_m} \bar{r}_m^{p_m} \le 1 \qquad & \forall m \in \mathcal{M} \qquad (30)\\
s_i^0 = 0, e_{km}^0 \qquad & \forall i \in \mathcal{N}, \forall k \in \mathcal{K}, \forall m \in \mathcal{M} \qquad (31)\\
y_{jkm}^t, r_{km}^t, e_k^t, s_i^t \in \mathbb{Z}_+, \bar{r}_m^{p_m} \in \{0,1\} \qquad & \forall i \in \mathcal{N}, \forall j \in \bar{\mathcal{N}}_{km}^t, \forall p_m \in \mathcal{P}_m,\\
& \forall k \in \mathcal{K}, \forall m \in \mathcal{M}, \forall t \in \mathcal{T}. \qquad (32)
\end{aligned}
$$

The objective function (27) minimises the inventory cost of objects and items, the cost of the production planning chosen in each period, and the waste cost. Constraints (28) and (29) are the operational demand and the tactical demand constraints. The convexity constraints are given in (30). Constraints (31) set the initial inventories to zero and constraints (32) define the domain of the variables.

To define the reduced cost of the subproblems associated to the production plans, consider the dual variables σ_{km}^t associated to constraints (29), and γ_m associated to (30), $\forall k \in \mathcal{K}$, $\forall m \in \mathcal{M}$ and $\forall t \in \mathcal{T}$. The objective function of the subproblem consists in reducing the production cost and the setup cost over the extreme points. Then, the subproblem related to machine m is given as follows.

$$
rc2_m = \text{minimise} \quad \sum_{t\in\mathcal{T}} \sum_{k\in\mathcal{K}} stc_{km} z_{km}^t + \sum_{t\in\mathcal{T}} \sum_{k\in\mathcal{K}} (c_{km}^t - \sigma_{km}^t) r_{km}^t - \gamma_t \qquad (33)
$$

$$
\begin{aligned}
\text{s. t.} \quad \sum_{k\in\mathcal{K}} (st_{km} z_{km}^t + vt_{km} r_{km}^t) \le Cap_m^t \quad & t \in \mathcal{T} \qquad (34)\\
r_{km}^t \le M_{km}^t z_{km}^t \quad & k \in \mathcal{K}, t \in \mathcal{T} \qquad (35)\\
r_{km}^t \in \mathbb{Z}_+, z_{km}^t \in \{0,1\} \quad & k \in \mathcal{K}, t \in \mathcal{T}. \qquad (36)
\end{aligned}
$$

The subproblem associated with the cutting stock problem is given by (19) - (21). In each iteration of the simplex method the subproblems (19) - (21) and (33) - (36) are solved. If $rc2_m \ge 0$ and $rc1_{km}^t \le 0$ for all k, m and t, then we have an optimal solution for the linear relaxation of the master problem (27) - (32).

The period decomposition model (PDM) can be equivalently described, where we have T subproblems and each one defines production plans for all machines in a period t. Thereby, in this case there are $K \times M \times T + T$ subproblems to be solved in each iteration of the simplex method.

Both decompositions provide the same lower bound, that is equal or better than the lower bound provided by the linear relaxation of the pattern-oriented model. To show this fact, let $\underline{z}^{POM}, \underline{z}^{PDM}$ and $\underline{z}^{MDM}$ be the optimal solution of the linear relaxation of pattern-oriented model, period decomposition model and machine decomposition model, respectively.

Proposition: $\underline{z}^{POM} \leq \underline{z}^{PDM} = \underline{z}^{MDM}$.

Proof: The inequality follows from the fact that applying the Dantzig-Wolfe decomposition, the lower bound of the decomposed problem can never be smaller than that one provided by the pattern-oriented model.

First of all, let us focus only on the columns obtained by the subproblems (34) - (36). The term that represents the value of these columns in the objective function (27) is given by:

$$\sum_{m \in \mathcal{M}} \sum_{p_m \in \mathcal{P}_m} c_m^{p_m} \bar{r}_m^{p_m}. \tag{37}$$

This is the only term that is different from the objective function of the master problem of the period decomposition, which is written as follows:

$$\sum_{t \in \mathcal{T}} \sum_{p_t \in \mathcal{P}_t} c_t^{p_t} \bar{r}_t^{p_t}, \tag{38}$$

where $c_t^{p_t} = \sum_{m \in \mathcal{M}} \sum_{k \in \mathcal{K}} stc_{km} z_{km}^t + \sum_{m \in \mathcal{M}} \sum_{k \in \mathcal{K}} c_{km}^t r_{km}^t$.

To prove the equality, consider the optimal solution $\underline{z}^{MDM}$ obtained by the machine decomposition model. For each subproblem m in (34) - (36), $m = 1, ..., M$, we represent the solution of a production plan p_m in the following matrix.

$$\begin{bmatrix} r_{1m}^1 & \dots & r_{Km}^1 & z_{1m}^1 & \dots & z_{Km}^1 \\ r_{1m}^2 & \dots & r_{Km}^2 & z_{1m}^2 & \dots & z_{Km}^2 \\ \vdots & \ddots & \vdots & \vdots & \ddots & \vdots \\ r_{1m}^T & \dots & r_{Km}^T & z_{1m}^T & \dots & z_{Km}^T \end{bmatrix} \tag{39}$$

where each line t $(t = 1, ..., T)$ of the matrix represents a feasible constraint in (34), i.e., $\sum_{k \in \mathcal{K}} (st_{km} z_{km}^t + vt_{km} r_{km}^t) \leq Cap_m^t$ for each $t = 1, ..., T$. The value of this solution in the master problem is given by:

$$\sum_{t \in \mathcal{T}} \sum_{k \in \mathcal{K}} (stc_{km} z_{km}^t + c_{km}^t r_{km}^t) \bar{r}_m^{p_m}. \tag{40}$$

By choosing the line t of a production plan p_m, $m = 1, ..., M$, we can compose a new matrix, that is given by:

$$\begin{bmatrix} r^t_{11} & \dots & r^t_{K1} & z^t_{11} & \dots & z^t_{K1} \\ r^t_{12} & \dots & r^t_{K2} & z^t_{12} & \dots & z^t_{K2} \\ \vdots & \ddots & \vdots & \vdots & \ddots & \vdots \\ r^t_{1M} & \dots & r^t_{KM} & z^t_{1M} & \dots & z^t_{KM} \end{bmatrix} \tag{41}$$

Note that:

- Matrix (41) represents a feasible solution of the t-th subproblem in the period decomposition, i.e., $\sum_{k\in\mathcal{K}}(st_{km}z^t_{km} + vt_{km}r^t_{km}) \leq Cap^t_m$ for $m = 1, ..., M$. Then, it represents a production plan p_t.
- By choosing the line t of a subset of subproblems and completing some lines of (41) with zero, it is possible to obtain more than one matrix of type (41). These matrices represents a subset of production plans $\tilde{\mathcal{P}}_t \subset \mathcal{P}_t$.
- The arrange of these production plans p_t must be done for all $t \in \mathcal{T}$ such way that the sum of their values must be equal to (40), i.e.:

$$\sum_{t\in\mathcal{T}}\sum_{p_t\in\tilde{\mathcal{P}}_t}(stc_{km}z^t_{km} + c^t_{km}r^t_{km})\bar{r}^{p_t}_t = \sum_{t\in\mathcal{T}}\sum_{k\in\mathcal{K}}(stc_{km}z^t_{km} + c^t_{km}r^t_{km})\bar{r}^{p_m}_m. \tag{42}$$

We have shown that a feasible solution of the machine decomposition model is also feasible for the period decomposition model. Then, $\underline{z}^{PDM} \leq \underline{z}^{MDM}$. Equivalently, we can show that $\underline{z}^{MDM} \leq \underline{z}^{PDM}$, that follows the equality. □

4 Computational results

In this section, we report our computational experience of the column generation methods. The tests were performed on an Intel Core i7 at 3.20 GHz and 16GB RAM. The methods were implemented in language C/C++. To solve the knapsack problems that determine cutting patterns we implemented a branch-and-bound method. The master problem and the subproblems in the lot sizing problem were solved by CPLEX 12.6. We have carried out computational experiments using instances of Poltroniere et al. (2008).

The linear relaxation of the master problems were solved by the column generation method. When an optimal solution has been found for the linear relaxation of the master problem, the integer restricted master problem with all columns is solved by CPLEX. To solve the integer restricted master problem, the computational time was limited to 600 seconds. In addition, we set the optimality gap tolerance to 0.2%.

The relaxation of the item-oriented model has not been investigated in this paper since it is well-known in the literature that relaxation of Kantorovich's model provides poor lower bounds.

Moreover, for big instances which we are dealing with, it is impossible to solve the item-oriented model.

4.1 Instances from literature

The instances of Poltroniere et al. (2008) are classified into 27 classes by varying the number of periods (T), of object grade types (K) and of item types for each grade type (n_k). There are two machines to produce the items ($M = 2$), where one machine produces objects of length $b_{k1} = 540$ cm, and the other one produces objects of $b_{k2} = 460$ cm. For each item, the length was generated randomly such that $l_i \in [0.1, 0.3]. \sum_{m \in \mathcal{M}} b_{km}/M$ for all $k \in \mathcal{K}$, and the demand is given by $d_i^t \in [0, 300]$ for all $t \in \mathcal{T}$. If $d_i^t < 50$ then $d_i^t = 0$. For each class, there are 10 instances. Further details on the instances can be found in Poltroniere et al. (2008).

4.2 Lower bounds

In this section, we compare the lower bound obtained by the column generation methods to solve the linear relaxation of pattern-oriented, period decomposition and machine decomposition models. The measure used to compare the methods were the average time to solve the linear relaxation, the number of iterations necessary to obtain an optimal solution, and the average gap. The gap is given by $100 * (\underline{z}^{DW} - \underline{z}^{POM})/\underline{z}^{DW}$, where $\underline{z}^{POM}$ is the lower bound of the pattern-oriented model and $\underline{z}^{DW}$ is the lower bound of the period or machine decomposition models.

As described in the previous section, the period and machine decomposition models provide the same lower bound. Then, the gap measures the deviation value of the period/machine decomposition and the pattern-oriented model. In this case, higher the gap, better is the lower bound provided by the decomposition models. For the instances from the literature, the lower bound of the decomposition models is slightly better than that one provided by the pattern-oriented model.

Three instances were proved by the column generation methods to be infeasible, each one in class 4, 7 and 16, respectively. The computational results are shown in Table 1, where for each formulation we report the average computational time in seconds (column Time(s)) and the average number of iterations for each class (column N. of iterations). Table 1 also shows the number of periods, classes, and number of items in each class (column 2). The last column shows the average gap between the pattern-oriented model and the period/machine decomposition models.

The computational time to solve the pattern-oriented model is always smaller than the computational time of the column generation method to solve the decomposition models. In addition, the computational time in the machine decomposition model is smaller for most classes compared to the period decomposition model. Only classes 10, 13, 16, 22 and 25 the computational time needed to solve the period decomposition model was smaller than the machine decomposition model. However, the number of iterations of the column generation method to solve the machine decomposition model is always higher. As expected when the instance size grows, the computational time and the number of iterations also grow.

Usually, the computational time to solve the linear relaxation of the machine decomposition model is smaller because there are only two subproblems associated with the lot sizing problem, while in the period decomposition model there are T subproblems, where $T \geq 8$. Solving the subproblems by CPLEX is reasonable fast, however, the time is increased by writing and sending the subproblem to CPLEX, and saving the optimal solution.

We can conclude that in terms of lower bound and computational time the column generation method performed on the machine decomposition model has a better performance over the remaining models. Despite its computational time being higher than the column generation method to solve pattern-oriented model, its lower bound has been shown slightly better than lower bound provided by the pattern-oriented model. This fact can be useful when the integer master problem is solved to optimality, as computational experiments have shown that CPLEX has some difficult to solve the integer restricted master problem to optimality.

Table 1: Computational performance of the algorithms to obtain the lower bounds.

		Compact Formulation		Period Decomposition		Machine Decomposition		
Class	T/K/n_k	Time(s)	N. of iteration	Time(s)	N. of iteration	Time(s)	N. of iteration	Gap(%)
1	8/2/5	0.34	7.50	11.72	8.70	2.79	29.10	0.011
2	8/2/10	1.12	19.50	28.45	20.90	4.75	44.80	0.003
3	8/2/20	2.98	37.70	61.66	38.40	8.16	43.50	0.003
4	10/2/5	0.43	8.00	12.57	8.56	4.23	41.89	0.008
5	10/2/10	1.29	18.10	25.16	20.60	5.64	48.30	0.011
6	10/2/20	3.94	38.50	59.06	38.00	15.27	53.80	0.004
7	12/2/5	0.46	7.89	17.03	8.67	11.29	61.00	0.020
8	12/2/10	1.74	21.40	39.28	21.10	11.93	66.40	0.005
9	12/2/20	5.04	37.30	79.47	36.90	14.95	64.80	0.004
10	8/4/5	0.58	8.60	67.04	13.20	70.87	79.10	0.003
11	8/4/10	2.09	19.60	103.75	21.40	70.99	82.00	0.001
12	8/4/20	7.74	40.00	148.60	41.50	41.56	42.00	0.000
13	10/4/5	0.75	9.10	89.10	12.40	188.74	125.10	0.006
14	10/4/10	2.93	20.40	117.49	20.80	94.20	117.00	0.002
15	10/4/20	11.71	40.00	169.70	42.10	95.02	47.70	0.002
16	12/4/5	0.90	8.89	84.27	12.44	217.20	160.67	0.003
17	12/4/10	4.02	20.50	124.79	22.10	101.20	160.50	0.000
18	12/4/20	14.69	40.40	208.82	41.90	109.12	52.30	0.003
19	8/6/5	0.79	9.20	117.63	17.90	121.98	139.80	0.001
20	8/6/10	3.80	21.80	172.98	22.50	132.75	119.50	0.000
21	8/6/20	14.40	41.00	224.12	41.10	120.05	54.70	0.002
22	10/6/5	1.13	8.90	141.54	18.50	224.83	209.00	0.004
23	10/6/10	6.11	21.70	149.45	22.50	64.54	60.50	0.001
24	10/6/20	20.47	41.00	216.89	42.20	112.73	59.50	0.001
25	12/6/5	1.38	8.40	146.97	18.10	325.99	346.90	0.002
26	12/6/10	8.18	21.90	175.58	22.60	77.47	71.90	0.000
27	12/6/20	27.97	42.40	258.14	42.10	145.08	75.30	0.000

4.3 Feasible Solutions

At the end of the column generation method, the variables of the restricted master problem are defined as integer and solved by CPLEX, where the computational time is limited to 600 seconds and the optimality gap of CPLEX is set to 0.2%. To evaluate the performance of the methods we consider the number of solved instances, the computational time, the percentage of lost of material and the gap. The gap is given by $100 * (z_{IP} - \underline{z})/z_{IP}$, where z_{IP} is the value of the optimal or incumbent solution of the restricted integer master problem, and $\underline{z}$ is the value of its linear relaxation obtained by the column generation method.

Table 2 shows the computational results obtained by CPLEX on the pattern-oriented model. Column Solved shows the number of solved instances, the average solution value is represented in column denoted by OBJ. Column %Lost shows the average lost of material, and column Gap is the average gap. Considering the cutting pattern generated in the column generation method, CPLEX was able to solve all instances that were proved to be feasible, except one instance in class 15 and one in class 26. For one instance in classes 4, 7 and 16, the linear relaxation of the master problem is infeasible. CPLEX can reach a solution in the given optimality gap in 600 seconds for most instances, as it shows the column Time(s) for the pattern-oriented model. Only 39 instances out of 265, that is proved to be feasible, CPLEX did not find a solution in the optimality gap of 0.2%. However, proving the optimality for these instances on the restricted integer master problem is a time-consuming process. The average of the optimality gap is very small (column Gap). If we consider the relative gap for each instance, the largest optimality gap is 0,49%. We have also run more difficult instances using CPLEX 12.4, that it did not find feasible solution, while CPLEX 12.6 found a feasible solution with gap smaller than 2,5%. In addition, the material lost due to cutting pattens performed is smaller than 1,1%. In a industry that we investigated, this lost is about 3%.

Table 2: Computational results of the pattern-oriented model to obtain upper bounds.

Class	Solved	OBJ	Time(s)	%Lost	Gap
1	10	49868.55	1.40	1.061	0.147
2	10	84127.49	4.01	0.121	0.162
3	10	174174.16	91.05	0.079	0.189
4	9*	57044.19	2.04	0.483	0.135
5	10	105901.79	12.56	0.152	0.166
6	10	220762.86	96.46	0.072	0.186
7	9*	64480.81	2.89	0.441	0.186
8	10	125735.67	46.76	0.122	0.161
9	10	264103.04	131.22	0.064	0.179
10	10	89622.28	3.89	0.785	0.168
11	10	183410.47	46.25	0.154	0.173
12	10	326957.17	301.56	0.083	0.210
13	10	112082.49	10.59	1.074	0.179
14	10	225668.09	82.65	0.131	0.195
15	9	417560.40	494.65	0.084	0.253
16	9*	135832.04	6.19	0.744	0.163
17	10	272148.53	47.31	0.251	0.176
18	10	499493.69	460.39	0.079	0.263
19	10	134475.56	10.51	1.019	0.198
20	10	267079.42	29.17	0.151	0.190
21	10	503950.26	462.00	0.111	0.317
22	10	169965.70	22.50	1.074	0.202
23	10	319623.65	165.66	0.131	0.191
24	10	635365.10	432.88	0.096	0.248
25	10	206779.63	30.17	0.658	0.210
26	9	383873.38	59.97	0.125	0.157
27	10	760539.74	438.67	0.096	0.280

* One instance out of 10 is proved to be infeasible.

We do not compare the objective values obtained by our methods with Poltroniere et al.

(2008), because the feasible regions can be different. Poltroniere et al. (2008) considered a set of additional constraints that use an estimative of the weight of waste occurred in the cutting process. This value is calculated in each iteration of their heuristics. However, the authors reported that the computational running time of the heuristics is not higher than 10 minutes. In addition, the pattern-oriented model solved 17 more instances than their best Lagrangian heuristic.

Table 3 shows the computational results of the period and machine decomposition. As in the pattern-oriented model, at the end of the column generation the variables of the integer restricted master problem were redefined as integer and solved by CPLEX. Then, to evaluate the methods we use the number of instances solved (column Solved), the average computational time (column Time(s)) and the average percentage of lost material in the cutting patterns usage (column %Lost). Considering the cutting patterns and the production plans generated in the column generation, CPLEX did not find feasible solution for classes 10 to 27. In addition, CPLEX found 77 feasible solutions on the period decomposition, and 80 on the machine decomposition model, out of 90 instances. The computational running time, the gap and the lost of material is higher than those obtained in the pattern-oriented model, mainly for the machine decomposition. Therefore, the strategy of solving the integer restricted master problem in the Dantzig-Wolfe decomposition has shown that the solution quality is considerable worse than solving the integer restricted master problem of the pattern-oriented model.

Table 3: Computational results of Dantzig-Wolfe decompositions to obtain upper bounds.

	Period Decomposition				Machine decomposition			
Class	Solved	Time(s)	%Lost	GAP	Solved	Time(s)	%Lost	GAP
1	8	12.72	3.592	6.737	8	77.99	26.636	18.003
2	8	39.39	2.748	7.533	10	6.51	4.959	22.581
3	10	85.74	0.977	10.268	9	18.98	2.187	18.937
4	6	115.43	2.321	6.862	9	6.02	13.376	22.850
5	10	39.35	2.085	6.601	9	8.77	7.064	20.019
6	10	254.34	0.929	5.943	10	33.48	1.715	16.916
7	6	26.87	2.896	5.653	6	15.91	10.516	21.649
8	9	98.39	1.545	3.690	9	14.00	4.427	14.313
9	10	485.94	0.554	5.540	10	97.99	1.263	15.234

We can conclude that performing the column generation on the cutting stock problem and solving the integer restricted master problem (pattern-oriented model) by CPLEX provides high quality solutions in a low computational time for the instances of the literature. In addition, the number of periods and items per class in these instances is adequate to meet the reality in industrial applications.

5 Conclusions

In this paper, we investigated column generation methods for the one-dimensional cutting stock problem integrated with lot sizing problem. The column generation was applied for linear relaxation of three mathematical models: pattern-oriented model, period decomposition model and machine decomposition model. The lower bound provided by the period and machine decomposition models is slightly better than the lower bound of the pattern-oriented model. After solving the relaxed master problem, we found an integer solution by defining the variables as integers. Computational results show that the pattern-oriented model gives almost optimal solution for all instances in the literature within low running time when it is solved by CPLEX 12.6.

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References

Arbib, C., Marinelli, F.. Integrating process optimization and inventory planning in cutting-stock with skiving option: An optimization model and its application. European Journal of Operational Research 2005;163(3):617 – 630.

Ben Amor, H., Valério de Carvalho, J.. Cutting stock problems. In: Column Generation; Springer Science + Business Media, 2005. p. 131–162.

Correia, M., Oliveira, J.F., Ferreira, J.. Reel and sheet cutting at a paper mill. Computers & Operations Research 2004;31(8):1223 – 1243.

Correia, M., Oliveira, J.F., Ferreira, J.S.. Integrated resolution of assignment, sequencing and cutting problems in paper production planning. International Journal of Production Research 2012;50(18):5195–5212.

Farley, A.. Mathematical-programming models for cutting-stock problems in the clothing industry. Journal of the Operational Research Society 1988;39(1):41–53.

Gilmore, P.C., Gomory, R.E.. A linear programming approach to the cutting-stock problem. Operations Research 1961;9(6):849–859.

Gramani, M., França, P., Arenales, M.. A lagrangian relaxation approach to a coupled lot-sizing and cutting stock problem. International Journal of Production Economics 2009;119(2):219 – 227.

Gramani, M.C.N., França, P.M.. The combined cutting stock and lot-sizing problem in industrial processes. European Journal of Operational Research 2006;174(1):509 – 521.

Gramani, M.C.N., França, P.M., Arenales, M.N.. A linear optimization approach to the combined production planning model. Journal of the Franklin Institute 2011;348(7):1523 – 1536.

Hendry, L., Fok, K., Shek, K.. A cutting stock and scheduling problem in the copper industry. Journal of the Operational Research Society 1996;47(1):38–47.

Kantorovich, L.V.. Mathematical methods of organizing and planning production. Management Science 1960;6(4):366–422.

Keskinocak, P., Wu, F., Goodwin, R., Murthy, S., Akkiraju, R., Kumaran, S., Derebail, A.. Scheduling solutions for the paper industry. Operations Research 2002;50(2):249–259.

Kim, K., Kim, B.I., Cho, H.. Multiple-choice knapsack-based heuristic algorithm for the two-stage two-dimensional cutting stock problem in the paper industry. International Journal of Production Research 2014;52(19):5675–5689.

Krichagina, E.V., Rubio, R., Taksar, M.I., Wein, L.M.. A dynamic stochastic stock-cutting problem. Operations Research 1998;46(5):690–701.

Menon, S., Schrage, L.. Order allocation for stock cutting in the paper industry. Operations Research 2002;50(2):324–332.

Nonås, S.L., Thorstenson, A.. A combined cutting-stock and lot-sizing problem. European Journal of Operational Research 2000;120(2):327 – 342.

Nonås, S.L., Thorstenson, A.. Solving a combined cutting-stock and lot-sizing problem with a column generating procedure. Computers & Operations Research 2008;35(10):3371 – 3392.

Poltroniere, S.C., Poldi, K.C., Toledo, F.M.B., Arenales, M.N.. A coupling cutting stock-lot sizing problem in the paper industry. Annals of Operations Research 2008;157(1):91–104.

Reinders, M.. Cutting stock optimization and integral production planning for centralized wood processing. Mathematical and Computer Modelling 1992;16(1):37 – 55.

Respicio, A., Captivo, M., Rodrigues, A.. A dss for production planning and scheduling in the paper industry. In: International Conference On Decision Making and Decision Support in the Internet Age. University College Cork, Cork; 2002. p. 298–308.

Santos, M.O., Almada-Lobo, B.. Integrated pulp and paper mill planning and scheduling. Computers & Industrial Engineering 2012;63(1):1 – 12.

Silva, E., Alvelos, F., Valerio, C.J.M.. Integrating two-dimensional cutting stock and lot-sizing problems. Journal of the Operational Research Society 2014;65(1):108–123.

Vance, P.. Branch-and-price algorithms for the one-dimensional cutting stock problem. Computational Optimization and Applications 1998;9(3):211–228.