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UNIFORM STABILITY OF A NON-AUTONOMOUS SEMILINEAR BRESSE SYSTEM WITH MEMORY

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Uniform stability of a non-autonomous semilinear Bresse system with memory

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Abstract

The Bresse system is a recognized mathematical model for vibrations of a circular arched beam that contains the class of Timoshenko beams when the arch's curvature is zero. It turns out that the majority of mathematical analysis to Bresse systems are concerned with the asymptotic stability of linear homogeneous problems. Under this scenario, we consider a nonlinear Bresse system modeling arched beams with memory effects, in a nonlinear elastic foundation. Then we establish uniform decay rates of the energy under time-dependent external forces.

Keywords: Bresse system, energy decay, visco-elasticity, infinite memory.

1 Introduction

In recent years, the Bresse system [4, 14] was studied by many authors. It is a robust mathematical model for vibrations of circular arched beams given by a system of three specially coupled wave equations. Let the variables φ, ψ, w, represent, respectively, vertical displacement, shear angle and axial displacement. Then the Bresse system can be deduced from the governing equations

$$\rho_1 \varphi_{tt} = Q_x + \ell N + F_1, \tag{1.1}$$
$$\rho_2 \psi_{tt} = M_x - Q + F_2, \tag{1.2}$$
$$\rho_1 w_{tt} = N_x - \ell Q + F_3, \tag{1.3}$$

together with the constitutive laws

$$N = k_0(w_x - \ell\varphi), \quad Q = k(\varphi_x + \ell w + \psi), \quad M = b\psi_x, \tag{1.4}$$

where Q, M, N stand for, respectively, shear force, bending moment, axial force, and $\ell > 0$ is the beam's curvature. The quantities $\rho_1, \rho_2, k, b, k_0$, are positive parameters of the system,

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and F_1, F_2, F_3 represent forcing terms and dissipative effects to the system. Inserting (1.4) into (1.1)-(1.3) we obtain the usual form of the Bresse system

$$\begin{aligned} \rho_1\varphi_{tt} - k(\varphi_x + \psi + \ell w)_x - k_0\ell(w_x - \ell\varphi) &= F_1, \\ \rho_2\psi_{tt} - b\psi_{xx} + k(\varphi_x + \psi + \ell w) &= F_2, \\ \rho_1 w_{tt} - k_0(w_x - \ell\varphi)_x + k\ell(\varphi_x + \psi + \ell w) &= F_3, \end{aligned}$$

defined in a bounded x-domain, say, $(0, L)$.

It is clear that when the parameter ℓ vanishes the Bresse system reduces to the Timoshenko system [1, 15, 22]

$$\begin{aligned} \rho_1\varphi_{tt} - k(\varphi_x + \psi)_x &= F_1, \\ \rho_2\psi_{tt} - b\psi_{xx} + k(\varphi_x + \psi) &= F_2, \end{aligned}$$

plus an independent wave equation $\rho_1 w_{tt} - k_0 w_{xx} = F_3$.

We recall that the Timoshenko system has a characteristic property related to the equal wave speeds condition. Indeed, with damping term present in only one of its equations, it is known that the energy of the Timoshenko system is exponentially stable if and only if

$$\frac{k}{\rho_1} = \frac{b}{\rho_2}$$

holds (e.g. [20, 2, 3, 13]). It happens that Bresse system also has a characteristic property related to the equal wave speeds condition, with further $k = k_0$. To this concern we refer the reader to, for instance, [1, 7, 8, 9, 17, 19, 21, 23].

Our study is related to the one by Guesmia and Kafini [12] where it was studied a Bresse system with infinite memory. More precisely, with notation $(g * u)(t) = \int_0^\infty g(s)u(t-s)ds$, they considered (1.1)-(1.3) with F_i as memory terms of the form

$$F_1 = -g_1 * \varphi_{xx}, \quad F_2 = -g_2 * \psi_{xx}, \quad F_3 = -g_3 * w_{xx},$$

where $g_i > 0$ are decreasing memory kernels. In a configuration with Dirichlet boundary condition and with prescribed past history for $\varphi(t), \psi(t), w(t)$, $t \le 0$, they studied the exponential and polynomial energy decay, in a history setting. Since the system has damping terms in all of the three equations, it was not assumed any equal wave speeds condition.

We notice that all above mentioned studies on Bresse systems deal with linear homogeneous problems. Dynamics of nonlinear Bresse systems was only recently studied in [15], where it was considered an autonomous problem without memory terms.

Here we study a Bresse system with memory in a framework with nonlinear foundation. Then our problem reads as follows:

$$\rho_1\varphi_{tt} - k(\varphi_x + \psi + \ell w)_x - k_0\ell(w_x - \ell\varphi) + g_1 * \varphi_{xx} + f_1(\varphi, \psi, w) = h_1, \tag{1.5}$$

$$\rho_2\psi_{tt} - b\psi_{xx} + k(\varphi_x + \psi + \ell w) + g_2 * \psi_{xx} + f_2(\varphi, \psi, w) = h_2, \tag{1.6}$$

$$\rho_1 w_{tt} - k_0(w_x - \ell\varphi)_x + k\ell(\varphi_x + \psi + \ell w) + g_3 * w_{xx} + f_3(\varphi, \psi, w) = h_3, \tag{1.7}$$

where $h_i = h_i(x,t)$ and $(f_1, f_2, f_3) = \nabla F$, for some potential function F. The needed prescribed past history is denoted by

$$\varphi(x,t) = \varphi_0(x,t), \quad \psi(x,t) = \psi_0(x,t), \quad w(x,t) = w_0(x,t), \quad t \leq 0, \quad x \in (0,L).$$

Our main objective is to prove the uniform stability of the system (1.5)-(1.7) under the influence of external time-dependent forces $h_i = h_i(x,t)$. The global existence for the system is presented in Theorem 2.1. The uniform exponential decay of the energy is presented in Theorem 2.2. To our best knowledge this is the first work concerned with Bresse systems with non-autonomous forces. However, non-autonomous Timoshenko systems were earlier studied in [11, 16].

2 Preliminaries

2.1 History setting

In order to deal with infinite memory a standard procedure is the one by Dafermos [6]. To this end we shall follow that arguments and notations in [10, 12]. Accordingly, one defines the following new variable η_i, for $t, s \geq 0$,

$$\begin{aligned} \eta_1^t(x,s) &= \varphi(x,t) - \varphi(x,t-s), \\ \eta_2^t(x,s) &= \psi(x,t) - \psi(x,t-s), \\ \eta_3^t(x,s) &= w(x,t) - w(x,t-s), \end{aligned}$$

that account for the past history. Then we obtain

$$\begin{aligned} \partial_t \eta_1^t(x,s) &= \varphi_t(x,t) - \partial_s \eta_1^t(x,s), \\ \partial_t \eta_2^t(x,s) &= \psi_t(x,t) - \partial_s \eta_2^t(x,s), \\ \partial_t \eta_3^t(x,s) &= w_t(x,t) - \partial_s \eta_3^t(x,s). \end{aligned}$$

From this, the memory terms become

$$\begin{aligned} \int_0^\infty g_1(s)\varphi_{xx}(t-s)ds &= -\int_0^\infty g_1(s)\partial_{xx}\eta_1^t(s)\,ds + g_1^0\,\varphi_{xx}(t), \\ \int_0^\infty g_2(s)\psi_{xx}(t-s)ds &= -\int_0^\infty g_2(s)\partial_{xx}\eta_2^t(s)\,ds + g_2^0\,\psi_{xx}(t), \\ \int_0^\infty g_3(s)w_{xx}(t-s)ds &= -\int_0^\infty g_3(s)\partial_{xx}\eta_3^t(s)\,ds + g_3^0\,w_{xx}(t), \end{aligned}$$

where $g_i \geq 0$ and

$$g_i^0 = \int_0^\infty g_i(s)\,ds > 0, \quad i = 1, 2, 3,$$

are assumed to be small. Finally, we obtain the new system (of six equations):

$$\rho_1\varphi_{tt} - k(\varphi_x + \psi + \ell w)_x - k_0\ell(w_x - \ell\varphi) - \int_0^\infty g_1\,\partial_{xx}\eta_1\,ds + g_1^0\,\varphi_{xx} + f_1 = h_1, \tag{2.8}$$

$$\rho_2\psi_{tt} - b\psi_{xx} + k(\varphi_x + \psi + \ell w) - \int_0^\infty g_2\,\partial_{xx}\eta_2\,ds + g_2^0\,\psi_{xx} + f_2 = h_2, \tag{2.9}$$

$$\rho_1 w_{tt} - k_0(w_x - \ell\varphi)_x + k\ell(\varphi_x + \psi + \ell w) - \int_0^\infty g_3\,\partial_{xx}\eta_3\,ds + g_3^0\,w_{xx} + f_3 = h_3, \tag{2.10}$$

$$\partial_t\eta_1 - \varphi_t + \partial_s\eta_1 = 0, \tag{2.11}$$

$$\partial_t\eta_2 - \psi_t + \partial_s\eta_2 = 0, \tag{2.12}$$

$$\partial_t\eta_3 - w_t + \partial_s\eta_3 = 0, \tag{2.13}$$

defined for $(x,t) \in (0,L) \times \mathbb{R}^+$, where $f_i = f_i(\varphi,\psi,w)$. To the system we add the boundary condition

$$\varphi(0,t) = \varphi(L,t) = \psi(0,t) = \psi(L,t) = w(0,t) = w(L,t) = 0, \;\; t \geq 0, \tag{2.14}$$

$$\eta_i^t(0,s) = \eta_i^t(L,s) = 0, \quad t,s \geq 0, \;\; i = 1,2,3, \tag{2.15}$$

and the initial conditions

$$\varphi(x,0) = \varphi_0(x), \quad \psi(x,0) = \psi_0(x), \quad w(x,0) = w_0(x), \quad x \in (0,L), \tag{2.16}$$

$$\varphi_t(x,0) = \varphi_1(x), \quad \psi_t(x,0) = \psi_1(x), \quad w_t(x,0) = w_1(x), \quad x \in (0,L), \tag{2.17}$$

$$\eta_i^0(x,s) = \eta_{i0}(x,s), \quad x \in (0,L), \;\; s > 0, \;\; i = 1,2,3, \tag{2.18}$$

with

$$\eta_i^t(x,0) = 0, \quad t \geq 0, \;\; x \in (0,L), \;\; i = 1,2,3. \tag{2.19}$$

In what follows we use standard Lebesgue space $L^p(0,L)$ and Sobolev spaces $H_0^1(0,L)$ and $H^2(0,L)$, with norm notation

$$\|u\|_p = \|u\|_{L^p} \quad \text{and} \quad \|u_x\|_2 = \|u\|_{H_0^1}.$$

The energy space to the system is defined by

$$\mathcal{H} = H_0^1(0,L)^3 \times L^2(0,L)^3 \times M_1 \times M_2 \times M_3,$$

where

$$M_i = L^2_{g_i}(\mathbb{R}^+; H_0^1(0,L)) = \left\{ \eta : \mathbb{R}^+ \to H_0^1(0,L) \;\Big|\; \int_0^\infty g_i(s)\|\partial_x\eta(s)\|_2^2\,ds < \infty \right\},$$

with norm

$$\|\eta\|_{M_i}^2 = \int_0^\infty g_i(s)\|\partial_x\eta(s)\|_2^2\,ds, \quad i = 1,2,3.$$

Given $z = (\varphi, \psi, w, \tilde{\varphi}, \tilde{\psi}, \tilde{w}, \eta_1, \eta_2, \eta_3) \in \mathcal{H}$, the usual norm is

$$\|z\|_{\text{usual}}^2 = \|\varphi_x\|_2^2 + \|\psi_x\|_2^2 + \|w_x\|_2^2 + \|\tilde{\varphi}\|_2^2 + \|\tilde{\psi}\|_2^2 + \|\tilde{w}\|_2^2 + \|\eta_1\|_{M_1}^2 + \|\eta_2\|_{M_2}^2 + \|\eta_3\|_{M_3}^2.$$

It is well known (cf. [15, 18]) that

$$\|(\varphi, \psi, w)\|_H^2 = \|\varphi_x\|_2^2 + \|\psi_x\|_2^2 + \|w_x\|_2^2$$

and

$$\|(\varphi, \psi, w)\|_B^2 = k\|\varphi_x + \psi + \ell w\|_2^2 + b\|\psi_x\|_2^2 + k_0\|w_x - \ell\varphi\|_2^2$$

are equivalent norms in $H_0^1(0, L)^3$. In particular, there exists $\gamma_B > 0$ such that

$$\|(\varphi, \psi, w)\|_B^2 \geq \gamma_B \|(\varphi, \psi, w)\|_H^2. \tag{2.20}$$

Then, if g_i^0 are sufficiently small, it follows that the Bresse norm

$$\begin{aligned} \|z\|_{\mathcal{H}}^2 = k\|\varphi_x + \psi + \ell w\|_2^2 + b\|\psi_x\|_2^2 + k_0\|w_x - \ell\varphi\|_2^2 + \rho_1\|\tilde{\varphi}\|_2^2 + \rho_2\|\tilde{\psi}\|_2^2 + \rho_1\|\tilde{w}\|_2^2 \\ + \|\eta_1\|_{M_1}^2 + \|\eta_2\|_{M_2}^2 + \|\eta_3\|_{M_3}^2 - g_1^0\,\|\varphi_x\|_2^2 - g_2^0\,\|\psi_x\|_2^2 - g_3^0\,\|w_x\|_2^2, \end{aligned} \tag{2.21}$$

is well-defined in $\mathcal{H}$ and equivalent to the usual one.

2.2 Assumptions

With respect to the kernel terms we assume, for each $i = 1, 2, 3$, $g_i \in C^0([0, \infty)) \cap C^1(\mathbb{R}^+)$,

$$g_i \geq 0, \quad g_i^0 = \int_0^\infty g_i(s)\,ds > 0, \quad g^0 = \max\{g_1^0, g_2^0, g_3^0\} < \gamma_B, \tag{2.22}$$

and for some $\xi > 0$,

$$g_i'(s) \leq -\xi g_i(s), \quad \forall s > 0. \tag{2.23}$$

With respect to the nonlinear foundation, we assume that there exists a function $F : \mathbb{R}^3 \to \mathbb{R}$, of class C^2, such that

$$\nabla F = (f_1, f_2, f_3), \tag{2.24}$$

and satisfying

$$\nabla F(\varphi, \psi, w)(\varphi, \psi, w) \geq F(\varphi, \psi, w) \geq 0, \tag{2.25}$$

and for some $p \geq 0$, there exists $C_F > 0$ such that,

$$|\nabla f_i(\varphi, \psi, w)| \leq C_F\big(1 + |\varphi|^p + |\psi|^p + |w|^p\big), \quad i = 1, 2, 3. \tag{2.26}$$

Finally, for the non-autonomous forcing, we assume that

$$h_i \in L^2_{\text{loc}}(\mathbb{R}^+, L^2(0, L)), \quad i = 1, 2, 3, \tag{2.27}$$

and there exist constants $\sigma, C_h > 0$ such that

$$\int_0^\infty e^{\sigma s}\big(\|h_1(s)\|_2^2 + \|h_2(s)\|_2^2 + \|h_3(s)\|_2^2\big)ds < C_h. \tag{2.28}$$

Examples of such (f_1, f_2, f_3) can be found in [15].

2.3 Results

The first result is dedicated to the global solvability.

Theorem 2.1. *Under the assumptions* (2.22)-(2.27)*, the Bresse system* (2.8)-(2.19) *has a unique weak solution*

$$z \in C^0([0,\infty);\mathcal{H}), \quad z(0) = z_0,$$

for any $z_0 \in \mathcal{H}$*. Moreover, if* $z_0 \in D(\mathcal{A})$ *and* $h_i \in H^1_{\text{loc}}([0,\infty);L^2(0,L))$*,* $i = 1,2,3$*, then the solution has regularity*

$$z \in C^1([0,\infty);\mathcal{H}) \cap C^0([0,\infty);D(\mathcal{A})).$$

The second result is dedicated to the decay of the energy. We note that from assumption (2.22) and inequality (2.20), the Bresse norm (2.21) is well defined. Then, along a weak solution $z(t) = (\varphi(t),\psi(t),w(t),\varphi_t(t),\psi_t(t),w_t(t),\eta_1^t,\eta_2^t,\eta_3^t)$, $t \geq 0$, the energy of the system is defined by

$$E(t) = \frac{1}{2}\|z(t)\|^2_{\mathcal{H}} + \int_0^L F(\varphi(t),\psi(t),w(t))\,dx.$$

In the next theorem, the uniform stability will require that the nonlinear terms f_i have at most a linear growth.

Theorem 2.2. *Under the assumptions* (2.22)-(2.28)*, with* $p = 0$ *in* (2.26)*, the energy of the Bresse system* (2.8)-(2.19) *has uniform exponential decay. More precisely,*

$$E(t) \leq C_0(E(0) + C_h)e^{-\gamma t}, \quad t \geq 0, \tag{2.29}$$

where $0 < \gamma \leq \sigma$ *and* $C_0 > 0$ *do not depend on the initial energy.*

In the case f_i are superlinear, we still have exponential decay of the energy, provided that $h_i = 0$.

Theorem 2.3. *Under the assumptions* (2.22)-(2.26)*, the energy of the Bresse system* (2.8)-(2.19)*, with* $h_i = 0$*, decays exponentially. More precisely, given* $R > 0$*, there exist constants* $C_R, \gamma_R > 0$ *such that*

$$E(t) \leq C_R E(0) e^{-\gamma_R t}, \quad t \geq 0, \tag{2.30}$$

for any solution z *with initial value satisfying* $\|z_0\|_{\mathcal{H}} \leq R$*.*

3 Global existence

In order to use semigroup theory we write our system (2.8)-(2.19) as a Cauchy problem

$$\frac{d}{dt}z(t) = \mathcal{A}z(t) + \mathcal{F}(t,z(t)), \quad z(0) = z_0, \tag{3.31}$$

where

$$z(t) = (\varphi(t),\psi(t),w(t),\varphi'(t),\psi'(t),w'(t),\eta_1^t,\eta_2^t,\eta_3^t) \in \mathcal{H}, \quad \varphi' = \varphi_t,\ \psi' = \psi_t,\ w' = w_t.$$

and initial data

$$z_0 = (\varphi_0, \psi_0, w_0, \varphi_0', \psi_0', w_0', \eta_{10}, \eta_{20}, \eta_{30}) \in \mathcal{H},$$

The operator $\mathcal{A}$ is linear and defined by

$$\mathcal{A}z = \begin{bmatrix} \varphi' \\ \psi' \\ w' \\ \frac{k}{\rho_1}(\varphi_x + \psi + \ell w)_x + \frac{k_0}{\rho_1}\ell(w_x - \ell\varphi) + \frac{1}{\rho_1}\int_0^\infty g_1(s)\partial_{xx}\eta_1(s)\,ds - \frac{g_1^0}{\rho_1}\varphi_{xx} \\ \frac{b}{\rho_2}\psi_{xx} - \frac{k}{\rho_2}(\varphi_x + \psi + \ell w) + \frac{1}{\rho_2}\int_0^\infty g_2(s)\partial_{xx}\eta_2(s)\,ds - \frac{g_2^0}{\rho_2}\psi_{xx} \\ \frac{k_0}{\rho_1}(w_x - \ell\varphi)_x - \frac{k}{\rho_2}\ell(\varphi_x + \psi + \ell w) + \frac{1}{\rho_1}\int_0^\infty g_3(s)\partial_{xx}\eta_3(s)\,ds - \frac{g_3^0}{\rho_1}w_{xx} \\ \varphi' - \partial_s\eta_1 \\ \psi' - \partial_s\eta_2 \\ w' - \partial_s\eta_3 \end{bmatrix},$$

with domain

$$D(\mathcal{A}) = \big\{ z \in \mathcal{H} \,|\, \mathcal{A}z \in \mathcal{H} \ \text{ and } \ \eta_i|_{s=0} = 0,\ i = 1,2,3 \big\}.$$

The nonlinear elastic foundation and non-autonomous forcing terms are given by $\mathcal{F} : [0,\infty) \times \mathcal{H} \to \mathcal{H}$,

$$\mathcal{F}(t,z) = \begin{bmatrix} 0 \\ 0 \\ 0 \\ \frac{1}{\rho_1}\big(h_1(t) - f_1(\varphi,\psi,w)\big) \\ \frac{1}{\rho_2}\big(h_2(t) - f_2(\varphi,\psi,w)\big) \\ \frac{1}{\rho_1}\big(h_3(t) - f_3(\varphi,\psi,w)\big) \\ 0 \\ 0 \\ 0 \end{bmatrix}.$$

The following energy estimate will be useful.

Lemma 3.1. *Given $\delta > 0$ one has*

$$\frac{d}{dt}E(t) \le \frac{1}{2}\sum_{i=1}^{3}\int_0^\infty g_i'(s)\|\partial_x\eta_i^t(s)\|_2^2\,ds + \delta\|(\varphi_t(t),\psi_t(t),w_t(t))\|_{(L^2)^3}^2 + \frac{1}{4\delta}\sum_{i=1}^{3}\|h_i(t)\|_2^2. \quad (3.32)$$

Proof. We shall assume the solution z regular, that is, $z \in C^0([0,\infty); D(\mathcal{A}))$. Then all the calculus below will be legitimate. The result also holds for weak solutions by a standard density argument.

Multiplying equations (2.8)-(2.10) by φ_t, ψ_t, w_t, respectively, and integrating over $(0, L)$ we conclude that

$$\frac{d}{dt}E = \frac{1}{2}\sum_{i=1}^{3}\int_0^\infty g_i' \|\partial_x \eta_i\|_2^2 \, ds + \int_0^L \big(h_1\varphi_t + h_2\psi_t + h_3 w_t\big) dx. \tag{3.33}$$

Then (3.32) follows. □

Proof of Theorem 2.1. The existence of solutions for the linear system $z_t = \mathcal{A}z$ was studied in [12] by showing that $-\mathcal{A}$ is maximal monotone in $\mathcal{H}$. To solve our nonlinear problem (3.31) we first observe that assumption (2.26) implies that $\mathcal{F}(t, z)$ is locally Lipschitz in z, for each fixed t . Indeed, let us take $z^1, z^2 \in \mathcal{H}$,

$$z^1 = (\varphi^1, \psi^1, w^1, \varphi^{1\prime}, \psi^{1\prime}, w^{1\prime}, \eta_1^1, \eta_2^1, \eta_3^1), \quad z^2 = (\varphi^2, \psi^2, w^2, \varphi^{2\prime}, \psi^{2\prime}, w^{2\prime}, \eta_1^2, \eta_2^2, \eta_3^2).$$

Now, using (2.26), there exists an embedding constant $C > 0$ such that

$$\begin{aligned}
&\frac{1}{\rho_1^2}\int_0^L |f_1(\varphi^1, \psi^1, w^1) - f_1(\varphi^2, \psi^2, w^2)|^2 \, dx \\
&\qquad \le C C_F\big(1 + \|z^1\|_{\mathcal{H}}^{2p} + \|z^2\|_{\mathcal{H}}^{2p}\big)\big(\|\varphi^1 - \varphi^2\|_2^2 + \|\psi^1 - \psi^2\|_2^2 + \|w^1 - w^2\|_2^2\big), \\
&\qquad \le C_r \|z^1 - z^2\|_{\mathcal{H}}^2,
\end{aligned}$$

where $C_r > 0$ depends on $r = \max\{\|z^1\|_{\mathcal{H}}, \|z^2\|_{\mathcal{H}}\}$. Same estimate holds for the cases f_2, f_3. Then

$$\|\mathcal{F}(t, z^1) - \mathcal{F}(t, z^2)\|_{\mathcal{H}} \le (3C_r)^{\frac{1}{2}}\|z^1 - z^2\|_{\mathcal{H}}, \quad t \ge 0,$$

which shows that $\mathcal{F}$ is locally Lipschitz. Hence from classical results, e.g., [5, Theorem 7.2], for $z_0 \in \mathcal{H}$, problem (2.8)-(2.19) has a unique weak solution $z = z(t)$ defined on an interval $[0, t_{\max})$. Moreover, if $t_{\max} < \infty$ then $\|z(t)\|_{\mathcal{H}} \to \infty$ as $t \to t_{\max}^-$. If the initial value $z_0 \in D(\mathcal{A})$, the same conclusion is valid for strong solutions.

It remains to show that the solution is global in time. From inequality (3.32) we infer that

$$E'(t) \le E(t) + \frac{1}{2}\sum_{i=1}^{3}\|h_i(t)\|_2^2, \quad t \in [0, t_{\max}).$$

Hence the Gronwall inequality implies that

$$E(t) \le \left(E(0) + \frac{1}{2}\sum_{i=1}^{3}\int_0^{t_{\max}} \|h_i(s)\|_2^2 \, ds\right) e^{t_{\max}}, \quad t \in [0, t_{\max}).$$

Since h_i are locally integrable we see that $E(t)$ is finite in $[0, t_{\max})$ whenever $t_{\max}$ is finite. Therefore $t_{\max} = \infty$ and this completes the proof of Theorem 2.1. □

4 Exponential stability

In this section we prove our main result by using energy methods. Let us define the functionals

$$I(t) = \int_0^L \rho_1\varphi(t)\varphi_t(t) + \rho_2\psi(t)\psi_t(t) + \rho_1 w(t)w_t(t)\,dx,$$
$$J_1(t) = -\rho_1 \int_0^\infty g_1(s)\int_0^L \eta_1^t(s)\varphi_t(t)\,dxds,$$
$$J_2(t) = -\rho_2 \int_0^\infty g_2(s)\int_0^L \eta_2^t(s)\psi_t(t)\,dxds,$$
$$J_3(t) = -\rho_1 \int_0^\infty g_3(s)\int_0^L \eta_3^t(s)w_t(t)\,dxds.$$

Denoting $J = J_1 + J_2 + J_3$ we consider the perturbed energy

$$\mathcal{L}(t) = E(t) + \varepsilon_2(\varepsilon_1 I(t) + J(t)), \quad t \geq 0,$$

where $\varepsilon_1, \varepsilon_2 > 0$ are parameters to be fixed later.

Lemma 4.1. *There exist constants $\varepsilon_0, \beta_1, \beta_2 > 0$ such that*

$$\beta_1 E(t) \leq \mathcal{L}(t) \leq \beta_1 E(t), \quad t \geq 0, \tag{4.34}$$

for any $\varepsilon_1, \varepsilon_2 \in (0, \epsilon_0]$.

Proof. Clearly there exists a constant $\tilde{C} > 0$ such that

$$|I(t)| + |J(t)| \leq \tilde{C}E(t), \quad t \geq 0.$$

Taking $\varepsilon_0 < \min\{1, \tilde{C}^{-1}\}$, we have

$$|\varepsilon_2(\varepsilon_1 I(t) + J(t))| < \varepsilon_0\tilde{C}E(t), \quad t \geq 0,$$

for $\varepsilon_1, \varepsilon_2 \leq \varepsilon_0$. Since $\varepsilon_0\tilde{C} < 1$, we obtain (4.34) with $\beta_1 = 1 - \varepsilon_0\tilde{C}$ and $\beta_2 = 1 + \varepsilon_0\tilde{C}$. □

Lemma 4.2. *There exist constants $\alpha, C_1, C_2, C_3 > 0$ such that*

$$\begin{aligned} I'(t) \leq &- E(t) - \alpha\|(\varphi_x(t), \psi_x(t), w_x(t))\|^2_{(L^2)^3} + C_1\|(\varphi_t(t), \psi_t(t), w_t(t))\|^2_{(L^2)^3} \\ &+ C_2\sum_{i=1}^3 \|h_i(t)\|_2^2 - C_3\sum_{i=1}^3\int_0^\infty g_i'(s)\|\partial_x\eta_i^t(s)\|_2^2\,ds, \quad t \geq 0. \end{aligned}$$

Proof. We begin by noting that

$$I' \leq \max\{\rho_1, \rho_2\}\|(\varphi_t, \psi_t, w_t)\|^2_{(L^2)^3} + \int_0^L \big(\rho_1\varphi_{tt}\varphi + \rho_2\psi_{tt}\psi + \rho_2 w_{tt}w\big)dx. \tag{4.35}$$

From the equations (2.8)-(2.13) we see that

$$\int_0^L \big(\rho_1\varphi_{tt}\varphi + \rho_2\psi_{tt}\psi + \rho_2 w_{tt}w\big)dx = -\,\|(\varphi,\psi,w)\|_B^2 + \big(g_1^0\|\varphi_x\|_2^2 + g_2^0\|\psi_x\|_2^2 + g_3^0\|w_x\|_2^2\big)$$
$$- \int_0^L \nabla F(\varphi,\psi,w)(\varphi,\psi,w)dx$$
$$+ \int_0^L \big(h_1\varphi + h_2\psi + h_3 w\big)dx + M,$$

where

$$M = -\int_0^\infty g_1 \int_0^L \partial_x\eta_1\,\varphi_x\,dx - \int_0^\infty g_2\int_0^L \partial_x\eta_2\,\psi_x\,dx - \int_0^\infty g_3 \int_0^L \partial_x\eta_3\,w_x\,dx.$$

For convenience, we can estimate

$$\int_0^L \big(h_1\varphi + h_2\psi + h_3 w\big)dx \le \frac{\gamma_B - g^0}{8}\|(\varphi_x,\psi_x,w_x)\|_{(L^2)^3}^2 + C_\gamma\sum_{i=1}^3\|h_i\|_2^2 \tag{4.36}$$

and

$$M \le \frac{\gamma_B - g^0}{8}\|(\varphi_x,\psi_x,w_x)\|_{(L^2)^3}^2 + C_\gamma\sum_{i=1}^3\|\eta_i\|_{M_i}^2, \tag{4.37}$$

for some $C_\gamma > 0$. Then, inserting (4.36)-(4.37) into (4.35) we obtain

$$\begin{aligned} I' \le &-\|(\varphi,\psi,w)\|_B^2 + \big(g_1^0\|\varphi_x\|_2^2 + g_2^0\|\psi_x\|_2^2 + g_3^0\|w_x\|_2^2\big) \\ &+ \frac{(\gamma_B - g^0)}{4}\|(\varphi_x,\psi_x,w_x)\|_{(L^2)^3}^2 + \max\{\rho_1,\rho_2\}\|(\varphi_t,\psi_t,w_t)\|_{(L^2)^3}^2 \\ &- \int_0^L \nabla F(\varphi,\psi,w)(\varphi,\psi,w)dx + C_\gamma\sum_{i=1}^3\|h_i\|_2^2 + C_\gamma\sum_{i=1}^3\|\eta_i\|_{M_i}^2. \end{aligned}$$

Adding $-E(t)$ to the inequality, and taking into account assumption (2.25), we obtain

$$\begin{aligned} I' \le &-E - \frac{1}{2}\|(\varphi,\psi,w)\|_B^2 + \frac{1}{2}\big(g_1^0\|\varphi_x\|_2^2 + g_2^0\|\psi_x\|_2^2 + g_3^0\|w_x\|_2^2\big) \\ &+ \frac{(\gamma_B - g^0)}{4}\|(\varphi_x,\psi_x,w_x)\|_{(L^2)^3}^2 + \frac{3}{2}\max\{\rho_1,\rho_2\}\|(\varphi_t,\psi_t,w_t)\|_{(L^2)^3}^2 \\ &+ C_\gamma\sum_{i=1}^3\|h_i\|_2^2 + \Big(\frac{1}{2} + C_\gamma\Big)\sum_{i=1}^3\|\eta_i\|_{M_i}^2. \end{aligned}$$

But using inequality (2.20) and assumption (2.22),

$$-\frac{1}{2}\|(\varphi,\psi,w)\|_B^2 + \frac{1}{2}\big(g_1^0\|\varphi_x\|_2^2 + g_2^0\|\psi_x\|_2^2 + g_3^0\|w_x\|_2^2\big) \le -\frac{(\gamma_B - g^0)}{2}\|(\varphi_x,\psi_x,w_x)\|_{(L^2)^3}^2.$$

Then, noting that,

$$\|\eta_i\|^2_{M_i} \leq -\frac{1}{\xi}\int_0^\infty g_i'\|\partial_x\eta_i\|_2^2\,ds, \quad i=1,2,3, \tag{4.38}$$

the lemma follows with $\alpha = \frac{1}{4}(\gamma_B - g^0)$, $C_1 = \frac{3}{2}\max\{\rho_1,\rho_2\}$, $C_2 = C_\gamma$ and $C_3 = \frac{1}{\xi}\big(\frac{1}{2}+C_\gamma\big)$. $\square$

Lemma 4.3. *Assume $p=0$ in (2.26). Then given $\nu>0$, there exists $C_\nu>0$ such that*

$$\begin{aligned} J'(t) \leq &-\kappa\|(\varphi_t(t),\psi_t(t),w_t(t))\|^2_{(L^2)^3} + \nu\|(\varphi_x(t),\psi_x(t),w_x(t))\|^2_{(L^2)^3} \\ &- C_\nu\sum_{i=1}^3\int_0^\infty g_i'(s)\|\partial_x\eta_i^t(s)\|_2^2\,ds + \sum_{i=1}^3\|h_i(t)\|_2^2, \quad t\geq 0, \end{aligned} \tag{4.39}$$

where $\kappa>0$ does not depend on ν.

Proof. We begin with

$$J_1' = A + B,$$

where

$$A = -\rho_1\int_0^\infty g_1\int_0^L \partial_t\eta_1\,\varphi_t\,dxds \quad\text{and}\quad B = -\int_0^L \rho_1\varphi_{tt}\int_0^\infty g_1\eta_1\,dsdx.$$

Since $\partial_t\eta_1 = \varphi_t - \partial_s\eta_1$, we have

$$A = -\rho_1 g_1^0\|\varphi_t\|_2^2 + \rho_1\int_0^\infty -g_1'\int_0^L \eta_1\,\varphi_t\,dxds.$$

Moreover, there exists $C_4>0$ such that

$$\begin{aligned} \rho_1\int_0^\infty -g_1'\int_0^L \eta_1\,\varphi_t\,dxds &\leq -C\rho_1\int_0^\infty g_1'\|\partial_x\eta_1\|_2\|\varphi_t\|_2\,ds \\ &\leq \frac{\rho_1 g_1^0}{2}\|\varphi_t\|_2^2 - C_4\int_0^\infty g_1'\|\partial_x\eta_1\|_2^2\,ds. \end{aligned}$$

Hence

$$A \leq -\frac{\rho_1 g_1^0}{2}\|\varphi_t\|_2^2 - C_4\int_0^\infty g_1'\|\partial_x\eta_1\|_2^2\,ds. \tag{4.40}$$

With respect to B, using equation (2.8), we obtain

$$B = \int_0^L\Big(-k(\varphi_x+\psi+\ell w)_x - k_0\ell(w_x-\ell\varphi) - \int_0^\infty g_1\partial_{xx}\eta_1\,ds + f_1 + g_1^0\varphi_{xx} - h_1\Big)\Big(\int_0^\infty g_1\eta_1\,ds\Big)dx.$$

We shall estimate each term of B. Clearly, given $\delta_1>0$ there exists $C_{\delta_1}>0$ such that,

$$\int_0^L -k(\varphi_x+\psi+\ell w)_x\int_0^\infty g_1\eta_1\,dsdx \leq \delta_1\|(\varphi_x,\psi_x,w_x)\|^2_{(L^2)^3} + C_{\delta_1}\|\eta_1\|^2_{M_1},$$

$$\int_0^L g_1^0\,\varphi_{xx}\int_0^\infty g_1\eta_1\,dsdx \leq \delta_1\|(\varphi_x,\psi_x,w_x)\|^2_{(L^2)^3} + C_{\delta_1}\|\eta_1\|^2_{M_1},$$

and

$$-\int_0^L k_0\ell(w_x-\ell\varphi)\int_0^\infty g_1\eta_1\,dsdx \le \delta_1\|(\varphi_x,\psi_x,w_x)\|^2_{(L^2)^3}+C_{\delta_1}\|\eta_1\|^2_{M_1}.$$

Also,

$$-\int_0^L\Big(\int_0^\infty g_1\partial_{xx}\eta_1\,ds\Big)\Big(\int_0^\infty g_1\eta_1\,ds\Big)dx \le g_1^0\|\eta_1\|^2_{M_1},$$

and

$$-\int_0^L h_1\int_0^\infty g_1\eta_1\,dsdx \le \|h_1\|_2^2+\frac{g_1^0\pi^2}{4L^2}\|\eta_1\|^2_{M_1}.$$

Finally,

$$\int_0^L f_1(\varphi,\psi,w)\int_0^\infty g_1\eta_1\,dsdx \le \frac{\pi}{L}\sqrt{g_1^0}\,\|f_1(\varphi,\psi,w)\|_2\|\eta_1\|_{M_1}.$$

But using (2.26) with $p=0$, there exists $C>0$ such that,

$$\|f_1(\varphi,\psi,w)\|_2^2 \le C\|(\varphi_x,\psi_x,w_x)\|^2_{(L^2)^3}.$$

Then, given $\delta_2>0$ there exists $C_{\delta_2}>0$ such that

$$\int_0^L f_1(\varphi,\psi,w)\int_0^\infty g_1\eta_1\,dsdx \le \delta_2\|(\varphi_x,\psi_x,w_x)\|^2_{(L^2)^3}+C_{\delta_2}\|\eta_1\|^2_{M_1}.$$

Combining above estimates, given $\delta_3>0$, there exists $C_{\delta_3}>0$ such that

$$B \le \delta_3\|(\varphi_x,\psi_x,w_x)\|^2_{(L^2)^3}+C_{\delta_3}\|\eta_1\|^2_{M_1}+\|h_1\|_2^2. \tag{4.41}$$

Then, from (4.40), (4.41) and (4.38), we conclude that, given $\delta'>0$ there exists $C_{\delta'}>0$ such that

$$J_1' \le -\frac{\rho_1 g_1^0}{2}\|\varphi_t\|_2^2+\delta'\|(\varphi_x,\psi_x,w_x)\|^2_{(L^2)^3}-C_{\delta'}\int_0^\infty g_1'\|\partial_x\eta_1\|_2^2\,ds+\|h_1\|_2^2.$$

Analogously we have,

$$J_2' \le -\frac{\rho_2 g_2^0}{2}\|\psi_t\|_2^2+\delta'\|(\varphi_x,\psi_x,w_x)\|^2_{(L^2)^3}-C_{\delta'}\int_0^\infty g_2'\|\partial_x\eta_2\|_2^2\,ds+\|h_2\|_2^2.$$

and

$$J_3' \le -\frac{\rho_1 g_3^0}{2}\|w_t\|_2^2+\delta'\|(\varphi_x,\psi_x,w_x)\|^2_{(L^2)^3}-C_{\delta'}\int_0^\infty g_3'\|\partial_x\eta_3\|_2^2\,ds+\|h_3\|_2^2.$$

Then we infer that given $\nu>0$ there exists $C_\nu>0$ such that

$$J' \le -\kappa\|(\varphi_t,\psi_t,w_t)\|^2_{(L^2)^3}+\nu\|(\varphi_x,\psi_x,w_x)\|^2_{(L^2)^3}-C_\nu\sum_{i=1}^3\int_0^\infty g_i'\|\partial_x\eta_i\|_2^2\,ds+\sum_{i=1}^3\|h_i\|_2^2,$$

where $\kappa=\frac{1}{2}\min\{\rho_1 g_1^0,\rho_2 g_2^0,\rho_1 g_3^0\}$. □

Proof of Theorem 2.2 The uniform decay of the energy follows from Lemmas 4.1-4.3 and energy estimate (3.32). Indeed, let us take $\varepsilon_1, \nu > 0$ such that

$$\varepsilon_1 C_1 < \frac{\kappa}{2} \quad \text{and} \quad \nu < \varepsilon_1 \alpha.$$

Then from Lemmas 4.2 and 4.3 we obtain

$$\begin{aligned} \varepsilon_1 I'(t) + J'(t) \leq & -\varepsilon_1 E(t) - \frac{\kappa}{2} \|(\varphi_t(t), \psi_t(t), w_t(t))\|^2_{(L^2)^3} \\ & + (C_2 + 1) \sum_{i=1}^{3} \|h_i(t)\|_2^2 - (C_3 + C_\nu) \sum_{i=1}^{3} \int_0^\infty g_i'(s) \|\partial_x \eta_i^t(s)\|_2^2 \, ds. \end{aligned}$$

Now, we choose $\varepsilon_2, \delta > 0$ such that

$$\varepsilon_2 (C_3 + C_\nu) < \frac{1}{2} \quad \text{and} \quad \delta < \varepsilon_2 \frac{\kappa}{2}.$$

Then from (3.32),

$$E'(t) + \varepsilon_2 \big(\varepsilon_1 I'(t) + J'(t)\big) \leq -\varepsilon_1 \varepsilon_2 E(t) + C_5 \sum_{i=1}^{3} \|h_i(t)\|_2^2,$$

where $C_5 = \frac{1}{4\delta} + C_2 + 1$. Choosing $\varepsilon_1, \varepsilon_2 \leq \varepsilon_0$, we infer from definition of $\mathcal{L}(t)$ and Lemma 4.1, that

$$\mathcal{L}'(t) \leq -\frac{\varepsilon_1 \varepsilon_2}{\beta_2} \mathcal{L}(t) + C_5 \sum_{i=1}^{3} \|h_i(t)\|_2^2, \quad t \geq 0. \tag{4.42}$$

Replacing $\frac{\varepsilon_1 \varepsilon_2}{\beta_2}$ in (4.42) by

$$\gamma = \min \Big\{ \frac{\varepsilon_1 \varepsilon_2}{\beta_2}, \sigma \Big\},$$

and using the integrand factor $e^{\gamma t}$, we see that

$$\mathcal{L}(t) \leq e^{-\gamma t} \mathcal{L}(0) + e^{-\gamma t} C_5 \int_0^t e^{\sigma s} \sum_{i=1}^{3} \|h_i(s)\|_2^2 \, ds.$$

Using Lemma 4.1 once more and assumption (2.28) we get

$$E(t) \leq \frac{\beta_2}{\beta_1} e^{-\gamma t} E(0) + \frac{1}{\beta_1} e^{-\gamma t} C_5 C_h,$$

which implies (2.29). We observe that positive constants β_1, β_2, C_5 do not depend on the initial energy. This completes the proof of Theorem 2.2. $\square$

Proof of Theorem 2.3 The arguments follow the same lines of the proof of Theorem 2.2. The main difference is in the Lemma 4.3, where now, the constant $C_\nu > 0$ in (4.39) is dependent on the initial data. Indeed, to prove Lemma 4.3 we need a estimate for $\|f_1(\varphi, \psi, w)\|_2^2$. Since $p > 0$ in (2.26), we have for some $C > 0$,

$$\begin{aligned}\|f_1(\varphi, \psi, w)\|_2^2 &\leq C \int_0^L \big(1 + |\varphi|^{2p} + |\psi|^{2p} + |w|^{2p}\big)\big(|\varphi|^2 + |\psi|^2 + |w|^2\big)\, dx \\ &\leq C\Big(1 + \|(\varphi_x, \psi_x, w_x)\|_{(L^2)^3}^{2p}\Big)\|(\varphi_x, \psi_x, w_x)\|_{(L^2)^3}^2 \\ &\leq C\big(1 + E(t)^p\big)\|(\varphi_x, \psi_x, w_x)\|_{(L^2)^3}^2.\end{aligned}$$

Now, because $h_i = 0$, identity (3.33) shows that energy $E(t)$ is decreasing and hence $E(t)^p \leq E(0)^p$, $t \geq 0$. In particular, for initial data satisfying $\|z_0\|_{\mathcal{H}} \leq R$, there exists $k_R > 0$ such that

$$\|f_1(\varphi, \psi, w)\|_2^2 \leq k_R\|(\varphi_x, \psi_x, w_x)\|_{(L^2)^3}^2.$$

Therefore, given $\delta_2 > 0$ there exists $C_{\delta_2} > 0$ (now dependent on R) such that

$$\int_0^L f_1(\varphi, \psi, w) \int_0^\infty g_1\eta_1\, ds dx \leq \delta_2\|(\varphi_x, \psi_x, w_x)\|_{(L^2)^3}^2 + C_{\delta_2}\|\eta_1\|_{M_1}^2,$$

and the rest of the proof of Lemma 4.3 remains unchanged.

To obtain (2.30) we follow the steps of the proof of Theorem 2.2 with $h_i = 0$ and taking into account that $C_\nu > 0$ in (4.39) depends on the initial data. □

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